Iwasawa algebras of p-adic Lie groups and Galois representations with open image

Jishnu Ray

2017-11-30

A key tool in the study of algebraic number fields are Iwasawa algebras, originally constructed by Iwasawa in the 1960s to study the "class groups" of fields, but since appearing in varied settings such as a Lazard's work on p-adic Lie groups and Fontaine's work on local Galois representations. For a prime p, the Iwasawa algebra of a p-adic Lie group G, denoted by $\mathbb{Z}_p[\![G]\!]$, is a non-commutative completed group algebra of G. In the first part of the talk, we lay the foundation by giving a very explicit description of certain Iwasawa algebras (one such algebra was described by my advisor Clozel). The base change map between the Iwasawa algebras over extensions of $\mathbb{Q}_p$ motivates us to discuss globally analytic p-adic representations following Emerton's work. In the second part of the talk, we will discuss about numerical experiments using a computer algebra system which give heuristic support to Greenberg's p-rationality conjecture which affirms the existence of "p-rational" number fields with Galois groups $(\mathbb{Z}/2\mathbb{Z})^t$. The p-rational fields are algebraic number fields whose Galois cohomology is particularly simple and which are interesting because they offer ways of constructing Galois representations with big open images. We go beyond Greenberg's work and construct novel Galois representations of the absolute Galois group of $\mathbb{Q}$ with big open images in reductive groups over $\mathbb{Z}_p$ (ex. $\mathrm{GL}(n;\mathbb{Z}_p)$; $\mathrm{SL}(n;\mathbb{Z}_p)$; $\mathrm{SO}(n;\mathbb{Z}_p)$; $\mathrm{Sp}(2n;\mathbb{Z}_p)$). We are proving results which show the existence of p-adic Lie extensions of $\mathbb{Q}$ where the Galois group corresponds to a certain specific p-adic Lie algebra (ex. $\mathfrak{sl}(n)$; $\mathfrak{so}(n)$; $\mathfrak{sp}(2n)$). This relates our work with a more general and classical Inverse Galois problem for p-adic Lie extensions.